Meson mass spectrum and OPE: matching to the large-N_c QCD*†

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Abstract

The relations between masses and decay constants of variety of meson resonances in the energy range 0–3 GeV are verified from the string-like, linear mass spectrum for vector, axial-vector, scalar and pseudoscalar mesons with a universal slope. The way to match the universality with the Operator Product Expansion (OPE) is proposed. The necessity of small deviations from linearity in parameterization of the meson mass spectrum and their decay constants is proven from matching to OPE.

As it follows from phenomenology [1, 2] the masses squared of mesons of a given spin increase linearly with the number of (radial) excitation n which can be thought of as a hint on the string structure of QCD. The string picture gives an equal slope of these trajectories for different quarkonium mesons since this quantity is proportional to the string tension depending on gluodynamics only. In our talk we examine possible corrections to the linear trajectories for vector (V), axial-vector (A), scalar (S), and pseudoscalar (P) case. Our method is based on the consideration of two-point correlators of quark currents in the large-N_c limit of QCD [3]. On one hand, by virtue of confinement they are saturated by an infinite set of narrow meson resonances, on the other hand their high-energy asymptotics are provided by the perturbation theory and the OPE [4] with condensates.

We adopt the following ansatz for the meson mass spectrum (compare to [5]):

$$m_R^2(n) = m_{0,R}^2 + a\,n + \frac{d_R}{n+1}\,, \tag{1}$$

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where $R \equiv V, A, S, P$ and the corrections to linear trajectory fit a possible deviation from the string picture in QCD. It turns out that for the consistency with the OPE one needs the following conditions on the residues:

$$\begin{aligned} Z_{VA}(n) &= t\left(m_{VA}^2(n)\right) \tfrac{dm_{VA}^2(n)}{dn}\,, \\ Z_{SP}(n) &= t\left(m_{SP}^2(n)\right) \left(m_{SP}^2(n) - \tfrac{3\alpha_s}{4\pi^3 C}\lambda^2\right) \tfrac{dm_{SP}^2(n)}{dn}\,; \end{aligned} \tag{2}$$

$$t\left(m_R^2(n)\right) = C + \sum_{i>0} A_i \exp\left(-B_i m_R^2(n)\right)\,, \qquad B_i > 0\,, \tag{3}$$

where λ represents the possible dimension two condensate. The wave function corrections in Eq. (3) are taken in the exponential form in order to avoid positive powers of logarithms of momentum in the high-energy asymptotics in accordance with OPE.

Our numerical calculations (see the tables at the end) have shown that:

1. The linear ansatz ($d_R = 0$) for the meson mass spectrum with a universal slope fits rather well the meson phenomenology [2] but for the corresponding residues (wave function moments) a related constant or linear ansatz ($A_i = 0$) in Eq. (2) leads to a striking disagreement with the OPE and, in particular, does not provide Chiral Symmetry Restoration (CSR) [6] , i.e. fast decreasing of differences between correlators of parity doublers ($S - P$, $V - A$) (compare to [7]).
2. Nonlinear corrections ($d_R \neq 0$) to the meson trajectories with universal slope may bring CSR but they do not improve drastically the matching to the OPE in each channel.
3. The OPE matching may be fulfilled with the help of exponentially decreasing corrections ($A_i \neq 0$) to the meson residues (wave function momenta), Eq. (3) (in our calculations we introduced two of such terms in each channel). They control substantially the values of condensates in high-energy asymptotics though being numerically negligible at low and intermediate energies.

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Table 1: An example of meson mass spectra. The inputs are: $a = (1200\,\mathrm{MeV})^2$, $\langle\bar{q}q\rangle = -(240\,\mathrm{MeV})^3$, $\langle\left(G^a_{\mu\nu}\right)^2\rangle = (360\,\mathrm{MeV})^4$, $\lambda = 0$ (this condensate has a tiny influence on the results), $f_\pi = 87\,\mathrm{MeV}$, $\alpha_s = 0.5$. The units are: $m(n)$, $F(n)$, m_0 — MeV; d — MeV^2; A_i — MeV^0; B_i — MeV^{-2}. The corresponding experimental values [8] are displayed in brackets.

Case	Inputs	Predictions
V	$m(0) = 770\,(769.3 \pm 0.8)$ $m(1) = 1460\,(1465 \pm 25)$ $F(0) = 150\,(154 \pm 8)$	$m_0 = 900$, $d = -470^2$ $m(2) = 1900\,(?)$, $m(3) = 2250\,(2149 \pm 17)$, ... $A_1 = 7 \cdot 10^{-4}$, $A_2 = -8 \cdot 10^{-10}$ $B_1 = 1000^{-2}$, $B_2 = 3160^{-2}$
A	$m(0) = 1200\,(1230 \pm 40)$ $m(1) = 1600\,(1640 \pm 40)$ $F(0) = 130\,(123 \pm 25)$	$m_0 = 910$, $d = 790^2$ $m(2) = 1970\,(?)$, $m(3) = 2290\,(?)$, ... $A_1 = -2$, $A_2 = 10^{-8}$ $B_1 = 1730^{-2}$, $B_2 = 14140^{-2}$
S	$m(0) = 900\,(980 \pm 10)$ $m(1) = 1500\,(1500 \pm 10)$ $F(0) = 320$	$m_0 = 910$, $d = -140^2$ $m(2) = 1920\,(?)$, $m(3) = 2260\,(2197 \pm 17)$, ... $A_1 = 22$, $A_2 = -1$ $B_1 = 2650^{-2}$, $B_2 = 1730^{-2}$
P π - on	$m(0) = 0\,(\approx 135)$ $m(1) = 1300\,(1300 \pm 100)$ $F(0) = 400$	$m_0 = 720$, $d = -720^2$ $m(2) = 1790\,(1801 \pm 13)$, $m(3) = 2160\,(?)$, ... $A_1 = 0.31$, $A_2 = -0.27$ $B_1 = 1730^{-2}$, $B_2 = 2240^{-2}$
P π - out	$m(0) = 1300\,(1300 \pm 100)$ $m(1) = 1800\,(1801 \pm 13)$ $F_\pi = 400$	$m_0 = 1390$, $d = -490^2$ $m(2) = 2170\,(?)$, $m(3) = 2480\,(?)$, ... $A_1 = -26$, $A_2 = 690$ $B_1 = 1730^{-2}$, $B_2 = 2240^{-2}$

Table 2: The values of residues for the first four states. In brackets we show the corresponding value without exponential corrections. Precision is ± 5 MeV.

Case	$F(0)$	$F(1)$	$F(2)$	$F(3)$
V	150 (160)	150 (150)	150 (150)	150 (150)
A	130 (110)	140 (140)	140 (140)	140 (140)
S	320 (430)	550 (560)	620 (630)	690 (690)
P π-on	400 (–)	540 (530)	620 (620)	670 (670)
P π-out	470 (520)	610 (610)	670 (670)	720 (720)